Surface Reasoning

Lecture 4: Negative Polarity and Antitonicity

Thomas Icard

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- In the previous lectures we focused on monotonicity *reasoning*, i.e. inferential patterns licensed by various functional words. We looked at polarity marking algorithms and proof systems for reasoning with containment relations at different types.
- One of the main themes is that a small amount of semantic information can be put into the syntax, so that these proof systems can be based merely on 'surface' syntactic information.
- However, there is a sense in which the 'semantic' information we have injected into the syntax should actually be part of the syntax already. Arguably, the best accounts of so-called negative polarity items (NPIs) make crucial reference to monotonicity and antitonicity.
- In this lecture we will first give a quick overview of NPIs, including some empirical work solidifying the connection between NPI distribution and antitonicity. Then we will look at several logical systems, extending the type-logical frameworks we have already seen, designed to capture aspects of NPI distribution.

- According to Giannakidou, in a survey article from last year [5], negative polarity items (NPIs) are characterized as expressions that cannot appear in a positive assertion with the simple past tense. The classic example is English 'any':
    - * Sue found any catfish.
- On the other hand, 'any' can appear in such contexts if it is within the scope of a negation:
    - Sue didn't find any catfish.
- Such expressions seem to occur in every documented language, with many interesting variations. Here we focus on English.

- NPIs seem to occur in English in multiple syntactic categories:
  - adverbs: 'ever', 'yet', 'one bit' ;
  - verb phrases: 'lift a finger', 'bat an eye' ;
  - noun phrases: 'a red cent' ;
  - prepositional phrases: 'in ages', 'in years' ;
  - determiner phrases: 'any', 'a single'.

- They also appear in many known contexts, apart from negation:
  - other 'n'-words: 'never', 'neither... nor'... ;
  - in restrictor/scope of quantifiers: 'no', 'every', 'not every', ... ;
  - antecedents of conditionals ;
  - comparative constructions ;
  - superlatives ;
  - non-factive verbs ;
  - questions ;
  - 'before', 'since', 'until' ;
  - 'only'.

- Note that with most of these expressions, and in most of these contexts, one can insert the word 'even' without affecting grammaticality.

- A common theme among most (though not obviously all) of these contexts is some kind of negativity, if not outright negation.
- Perhaps the most influential and long-standing proposal, due originally to Fauconnier and explored in much more depth by Ladusaw, is that, at least roughly, the crucial feature is *antitonicity*, which is in general much weaker than negation.
- There are uses of some of these expressions that do not function as NPIs. For instance, the following is not a counterexample to the Fauconnier/Ladusaw hypothesis:
  - Any first-year student could figure that out.

  This is sometimes called *free choice* 'any'. Some have tried to link the analysis of NPIs with that of free choice items (FCIs). We will see one formal example of how this can be done in what follows. A diagnostic for FCI 'any' is modification with 'almost'.

- Of those contexts listed on the previous slide, we have already seen that many of them are indeed antitone: 'no', 'every', 'not every', and antecedents of (material) conditionals. Many of the others are as well. For instance,

  That is the tallest building I have ever seen $\rightsquigarrow$
  That is the tallest brick building I have ever seen.

  That is, supposing the building I am seeing is a brick building.

- In fact, all of the other contexts above which seem to be antitone nonetheless have this caveat:

  Only Ella brought a tent. $\rightsquigarrow$
  Only Ella brought a two-person tent.

  The clock struck 12 before she made it to the ball. $\rightsquigarrow$
  The clock struck 12 before she made it to the ball and had a glass of wine.

Strawson Entailment

- In an influential paper von Fintel (1999) proposes we replace entailment by what he calls 'Strawson entailment', which takes into account presuppositions. The antitonicity condition there becomes:
  - If $x \leq y$ and $f(x)$ is defined, then $f(y) \leq f(x)$.
- The second two examples then become:

Only Ella brought a tent.
<u>Ella brought a two-person tent.</u>
Only Ella brought a two-person tent.

The clock struck twelve before she made it to the ball.
<u>She made it to the ball and had a glass of wine.</u>
The clock struck 12 before she made it to the ball and had a glass of wine.

- There are certainly many difficult and subtle issues here that should be understood.
- For the purpose of testing the Fauconnier/Ladusaw hypothesis, I suggest there is sometimes an easier method that bypasses some of these difficulties involving presupposition and other thorny issues. First note that the following lemma holds:

Lemma

The following are equivalent to f being antitone:

- $f(x \vee y) \leq f(x) \wedge f(y)$;
- $f(x) \vee f(y) \leq f(x \wedge y)$.

- Instead of checking for the usual definition of antitonicity (or monotonicity), it may be useful to capitalize on these equivalents:

Roger Bannister was the first athlete to run a sub 4:00 mile or to be named *Sports Illustrated* "Sportsman of the Year".

$\Rightarrow$ Roger Bannister was the first athlete to run a sub 4:00 mile, and he was the first athlete to be named "Sportsman of the Year".

If you put sugar or honey in your tea, it will taste sweet.

$\Rightarrow$ If you put sugar in your tea, it will taste sweet; and if you put honey in your tea, it will taste sweet.

- The truth is, neither of these strategies will save the analysis from all counterexamples. For instance, why are NPIs allowed in questions:

  - Do you have any sweet tea (at all)?
  - Is this lecture over yet?

  In what sense could be these antitone contexts?

- There are many proposals in the linguistics literature, each with its own strengths and weaknesses: theories based on domain widening, entropy, non-veridicality, pragmatic negation, and so on.

- For our purposes, it is enough that there is some close connection between negative polarity and antitonicity. Any successful account will have to explain why this connection holds. And the work described in the rest of this lecture demonstrates the fecundity of this idea as a rough starting point.

- In the linguistics and logic literatures, a large number of downward entailing / antitone contexts across a number of languages have been documented. See especially papers by Ladusaw and Lawler. As we will see tomorrow, and as you can probably already imagine, detecting such contexts is important for many NLU tasks.

- However, there are certainly many more antitone environments in English, and cross-linguistically cataloguing such items is impractical.

- The main insight of Danescu et al. (2009) is that NPIs can offer an efficient way of learning new antitone contexts in an unsupervised way. The basic idea is that if a word co-occurs with known NPIs significantly, then that word is likely to create an antitone context and support the corresponding inferences.

1. Choose a handful of well established NPIs:

    - any
    - at all
    - yet
    - do a thing
    - bat an eye
    - in ages
    - ever
    - take long
    - leave until
    - would mind
    - budge
    - red cent
    - eat a bite
    - bother to
    - lift a finger
    - to speak of
    - drink a drop
    - give a damn

2. Collect all the words w that appear to the left of an NPI up to the next punctuation mark. E.g. in '*By the way, we don't have plans anymore because they died*', we would take '*we don't have plans*'.
3. For each such word w, check whether:

$$\frac{c_{NPI}(w)}{\sum_{w' \in W} c_{NPI}(w')} > \frac{c(w)}{\sum_{w' \in W} c(w')}$$

   where $c_{NPI}(w)$ is the number of times w appears in an NPI context, and $c(w)$ is the count of w in the whole corpus.
4. If it is, w is a potential antitone functional word.

- In order to find new contexts, contexts are thrown out that contain the 10 most common: 'not', 'no', 'none', and so on.
- With this adjustment, the score of a word $S(w)$ is the ratio of the two weighted counts on the last side.
- To avoid collecting "piggybackers" like 'vigorously', which occurs frequently with 'deny' and 'oppose', Danescu et al. devise a distilled scoring function, punishing those that often appear in contexts with other high-scoring words:

$$S_d(w) = \frac{\sum_{\text{contexts } k} \frac{S(w)}{n(k)}}{N(w)},$$

where $n(k) = \sum_{w' \in k} S(w')$ and $N(w)$ is the number of NPI contexts containing w.

Results

- Precision was very high (around 80%), but most important for this task is recall.
- Impressively, the algorithm produced a number of novel words that had not appeared on previous lists:

absence of	to defer	hardly (**L**)	premature to	to rule out	to veto
absent from	to deny (**L**)	to lack	to prevent	skeptical •	wary of
anxious about •	to deter	innocent of •	to prohibit	to suspend	warned that (**L**)
to avoid (**L**)	to discourage	to minimize •	rarely (**L**)	to thwart	whenever
to bar	to dismiss	never (**L**)	to refrain from	unable to	withstand
barely	to doubt (**L**)	nobody	to refuse (**L**)	unaware of	
to block	to eliminate	nothing	regardless •	unclear on	
cannot (**L**)	essential for •	to oppose	to reject	unlike	
compensate for •	to exclude	to postpone •	reluctant to (**L**)	unlikely (**L**)	
to decline	to fail (**L**)	to preclude	to resist	unwilling to	

- Moral: The correspondence between antitonicity and licensing of NPIs is not illusory! Now back to logic.

- David Dowty proposed an alternative to van Benthem and Sánchez-Valencia's Monotonicity Calculus.

  "The goal is to 'collapse' the independent steps of Monotonicity Marking and Polarity Determination into the syntactic derivation itself, so that words and constituents are generated with the markings already in place that they would receive in Sánchez's polarity summaries." ([4], p.7)

- Since the main issue is to govern properly the distribution of NPIs, we cannot have the polarity determination coming after the grammaticality determination. Since NPIs are sensitive to polarity, this must all happen in tandem.

- In this presentation of Dowty's system I partly follow Moss' formulation [6], who uses marked types in place of marked categories. It can be done either way, however.

Internalized Type System

- The set of types $\mathcal{T}^* \supseteq \mathcal{T}$ is the smallest set such that:
  - Basic types $e, t \in \mathcal{T}^*$;
  - If $\sigma, \tau \in \mathcal{T}^*$, then $\sigma \rightarrow \tau \in \mathcal{T}^*$;
  - If $\tau \in \mathcal{T}^*$, then $\overline{\tau} \in \mathcal{T}^*$.
- Interpretation of typed terms is in the usual domains, except that all functional domains are assumed to contain only monotone functions:

$$\mathbb{M}_{\sigma\rightarrow\tau} = \{f \in M_\sigma^{M_\tau} \mid f \text{ is monotone}\} = [\mathbb{M}_\sigma, \mathbb{M}_\tau].$$

$$\mathbb{M}_{\overline{\tau}} = \mathbb{M}_\tau^{op}.$$

- This motivates the following equivalence relation $\simeq$ on $\mathcal{T}^*$:
  - $\overline{(\overline{\tau})} \simeq \tau$;
  - $\overline{\sigma \rightarrow \tau} \simeq \overline{\sigma} \rightarrow \overline{\tau}$;
  - If $\tau \simeq \tau'$, then $\overline{\tau} \simeq \overline{\tau'}$;
  - If $\sigma \simeq \sigma'$ and $\tau \simeq \tau'$, then $\sigma \rightarrow \tau \simeq \sigma' \rightarrow \tau'$.
- Clearly, if $\tau \simeq \sigma$, then $\mathbb{M}_\tau = \mathbb{M}_\sigma$. From here on we consider $\mathcal{T}^*_\simeq$.

- For a fragment of language, we must type each lexical entry at least twice. Here is a simple fragment. Again, we abbreviate $e \longrightarrow t$ by p.

  - Theodore: $\begin{cases} p \longrightarrow t \\ \overline{p} \longrightarrow \overline{t} \end{cases}$
  - every: $\begin{cases} \overline{p} \longrightarrow (p \longrightarrow t) \\ p \longrightarrow (\overline{p} \longrightarrow \overline{t}) \end{cases}$
  - some: $\begin{cases} p \longrightarrow (p \longrightarrow t) \\ \overline{p} \longrightarrow (\overline{p} \longrightarrow \overline{t}) \end{cases}$
  - no: $\begin{cases} \overline{p} \longrightarrow (\overline{p} \longrightarrow t) \\ p \longrightarrow (p \longrightarrow \overline{t}) \end{cases}$
  - candidate: $\begin{cases} p \\ \overline{p} \end{cases}$
  - proposal: $\begin{cases} p \\ \overline{p} \end{cases}$
  - who: $\begin{cases} p \longrightarrow (p \longrightarrow p) \\ \overline{p} \longrightarrow (\overline{p} \longrightarrow \overline{p}) \end{cases}$
  - likes: $\begin{cases} (p \longrightarrow t) \longrightarrow p \\ (\overline{p} \longrightarrow \overline{t}) \longrightarrow \overline{p} \\ (\overline{p} \longrightarrow t) \longrightarrow p \\ (p \longrightarrow \overline{t}) \longrightarrow \overline{p} \end{cases}$

- In this setting, for an expression to be grammatical, it must be provably of type t (type $\overline{t}$ does not count).

Including NPIs

- We can now introduce our first NPI:

$$\text{any} : \begin{cases} \overline{p} \longrightarrow (\overline{p} \longrightarrow \overline{t}) \\ \overline{p} \longrightarrow (p \longrightarrow t) \end{cases}$$

- Notice 'any' can behave like 'some' or 'every', as NPI or FCI.

  If they will charge any amount, they will charge any amount.

- The first is of type $\overline{p} \longrightarrow (\overline{p} \longrightarrow \overline{t})$, the second has type $\overline{p} \longrightarrow (p \longrightarrow t)$.

- We can also have other verbs that create antitone contexts:

$$\text{doesn't\_like} : \begin{cases} (p \longrightarrow t) \longrightarrow \overline{p} \\ (\overline{p} \longrightarrow \overline{t}) \longrightarrow p \end{cases}$$

- It is possible to check that the following are all grammatical:
  - Theodore doesn't like any proposal.
  - No candidate likes any proposal.
  - Every candidate who likes any candidate likes Theodore.
- The following are grammatical, but 'any' is a FCI here:
  - Theodore likes any proposal.
  - Any candidate likes Theodore.
  - Some candidate likes any proposal.
- Finally, 'any' can be both in the same sentence:
  - Every candidate who likes any proposal likes any proposal.
  - Any candidate who likes any proposal likes any proposal.
- While the following just means, 'No candidate likes any proposal':
  - No candidate who likes any proposal likes any proposal.

Example 1

$$\dfrac{\dfrac{\text{no:}\overline{p} \longrightarrow (\overline{p} \longrightarrow t) \qquad \text{candidate:}\overline{p}}{\text{no candidate: } \overline{p} \longrightarrow t} \qquad \dfrac{\text{likes:}(\overline{p} \longrightarrow \overline{t}) \longrightarrow \overline{p} \qquad \dfrac{\text{any:}\overline{p} \longrightarrow (\overline{p} \longrightarrow \overline{t}) \qquad \text{proposal:}\overline{p}}{\text{any proposal: } \overline{p} \longrightarrow \overline{t}}}{\text{likes any proposal: } \overline{p}}}{\text{no candidate likes any proposal: } t}$$

- Several things to notice:
  - The other lexical entry for 'any' would not make the entire expression of type t, but rather $\overline{t}$.
  - The polarity information for all nodes in the tree is already correct, and could be used in an inference system. E.g. 'candidate' is marked $-$, as is the constituent 'likes any proposal'.

Examples 2 and 3

$$\dfrac{\text{Theodore: } p \rightarrow t \qquad \dfrac{\text{likes: } (p \rightarrow t) \rightarrow p \qquad \dfrac{\text{any: } \overline{p} \rightarrow (p \rightarrow t) \qquad \text{proposal: } \overline{p}}{\text{any proposal: } p \rightarrow t}}{\text{likes any proposal: } p}}{\text{Theodore likes any proposal: } t}$$

$$\dfrac{\text{Theodore: } p \rightarrow t \qquad \dfrac{\text{doesn't\_like: } (\overline{p} \rightarrow \overline{t}) \rightarrow p \qquad \dfrac{\text{any: } \overline{p} \rightarrow (\overline{p} \rightarrow \overline{t}) \qquad \text{proposal: } \overline{p}}{\text{any proposal: } \overline{p} \rightarrow \overline{t}}}{\text{doesn't\_like any proposal: } p}}{\text{Theodore doesn't\_like any proposal: } t}$$

(N.B. 'Theodore doesn't like no candidate' is correctly(?) ruled out.)

Example 4

$$
\dfrac{
\dfrac{\text{some:}p \longrightarrow (p \longrightarrow t) \qquad \text{candidate:}p}{\text{some candidate: } p \longrightarrow t}
\qquad
\dfrac{\text{likes:}(\overline{p} \longrightarrow t) \longrightarrow p \qquad \dfrac{\text{no:}\overline{p} \longrightarrow (\overline{p} \longrightarrow t) \qquad \text{proposal:}\overline{p}}{\text{no proposal: } \overline{p} \longrightarrow t}}{\text{likes no proposal: } p}
}{\text{some candidate likes no proposal: } t}
$$

Examples 5, 6, and 7

$$\dfrac{\dfrac{\text{no}:\overline{p}\longrightarrow(\overline{p}\longrightarrow t)\qquad \text{candidate}:\overline{p}}{\text{no candidate}:\ \overline{p}\longrightarrow t}\qquad \dfrac{\text{likes}:(\overline{p}\longrightarrow\overline{t})\longrightarrow\overline{p}\qquad \dfrac{\text{every}:p\longrightarrow(\overline{p}\longrightarrow\overline{t})\qquad \text{proposal}:p}{\text{every proposal}:\ \overline{p}\longrightarrow\overline{t}}}{\text{likes every proposal}:\ \overline{p}}}{\text{no candidate likes every proposal}:\ t}$$

$$\dfrac{\dfrac{\text{no}:\overline{p}\longrightarrow(\overline{p}\longrightarrow t)\qquad \text{candidate}:\overline{p}}{\text{no candidate}:\ \overline{p}\longrightarrow t}\qquad \dfrac{\text{doesn't\_like}:(p\longrightarrow t)\longrightarrow\overline{p}\qquad \dfrac{\text{every}:\overline{p}\longrightarrow(p\longrightarrow t)\qquad \text{proposal}:\overline{p}}{\text{every proposal}:\ p\longrightarrow t}}{\text{doesn't\_like every proposal}:\ \overline{p}}}{\text{no candidate doesn't\_like every proposal}:\ t}$$

$$\dfrac{\dfrac{\text{no}:\overline{p}\longrightarrow(\overline{p}\longrightarrow t)\qquad \text{candidate}:\overline{p}}{\text{no candidate}:\ \overline{p}\longrightarrow t}\qquad \dfrac{\text{doesn't\_like}:(p\longrightarrow t)\longrightarrow\overline{p}\qquad \dfrac{\text{any}:\overline{p}\longrightarrow(p\longrightarrow t)\qquad \text{proposal}:\overline{p}}{\text{any proposal}:\ p\longrightarrow t}}{\text{doesn't\_like any proposal}:\ \overline{p}}}{\text{no candidate doesn't\_like any proposal}:\ t}$$

Moss [6] explores the system from a logical point of view. He shows that it provides for a very elegant version of the proof system we saw from Zamansky et al. The key feature is that we do not have to deal with markings at all in the axioms, since all functional types are assumed to be interpreted as monotone functions (sometimes with opposite pre-orders).

$$\text{(refl)}\ \frac{}{t \leq t} \qquad \text{(trans)}\ \frac{t \leq s \qquad s \leq u}{t \leq u}$$

$$\text{(mono)}\ \frac{u \leq v}{t(u) \leq t(v)}$$

$$\text{(repl)}\ \frac{t \leq s \qquad u \equiv v}{t(u) \leq s(v)} \qquad \text{(abstr)}\ \frac{t \leq s}{\lambda x.t \leq \lambda x.s}$$

$$(\beta)\ \frac{}{t[x/u] \equiv (\lambda x.t)(u)} \qquad (\eta)\ \frac{}{t \ \equiv \ \lambda x.t(x)}$$

- From a linguistic point of view, Dowty's system is also of interest.
- Dowty himself uses it to explore negative concord phenomena as well (e.g. 'ne ... pas' in French, etc.).
- Bernardi, however, finds several problems with it. For instance, Dowty incorrectly predicts that NPI 'anybody' should *not* be licensed here, whereas intuitively it is:

  ** If Theodore doubts anybody left, he won't vote.*

- This motivates Bernardi's own proposal based on multimodal categorial grammar.

- Raffaella Bernardi's framework for analyzing NPIs follows recent work in categorial type logics. In particular the framework is based on Moortgat's *Logic of Residuation*.

- We will first introduce residuation and the logic thereof (a topic of interest in its own right), then discuss the application to NPIs and monotonicity reasoning.

Definition (Residuation)

Suppose $\mathcal{A} = (A, \sqsubseteq_{\mathcal{A}})$ and $\mathcal{B} = (B, \sqsubseteq_{\mathcal{B}})$ are partial orders. Then the pair of $f : A \to B$ and $g : B \to A$ form a *residuated pair* just in case:

$$f(a) \sqsubseteq_{\mathcal{B}} b \text{ if and only if } a \sqsubseteq_{\mathcal{A}} g(b),$$

for all $a \in A$ and $b \in B$.

An equivalent definition is the following:

- f and g are monotone ;
- $f(g(b)) \sqsubseteq_{\mathcal{B}} b$;
- $a \sqsubseteq_{\mathcal{A}} g(f(a))$;

for all $a \in A$ and $b \in B$.

(If we take $\mathcal{B}^{op}$ instead, f and g form what is called a *Galois connection*.)

Simple Example

- Let $\mathcal{A} = \mathcal{B} = (\mathbb{Q}^+, \leq)$.
- Let $f(n) = n \times 7$ and $g(n) = n/7$.
- Clearly both f and g are monotone, and:

$$f(g(n)) = f(n/7) = (n/7) \times 7 = n$$

$$g(f(n)) = g(n \times 7) = (n \times 7)/7 = n.$$

- Alternatively,

$$\begin{aligned} f(n) \leq m \quad &\Leftrightarrow \quad n \times 7 \leq m \\ &\Leftrightarrow \quad n \leq m/7 \\ &\Leftrightarrow \quad n \leq g(m) \end{aligned}$$

- f and g form a residuated pair.

Another Example

- Consider the modal operators $\Diamond$ and $\Box^{\downarrow}$, where in a modal model $[\![\Box^{\downarrow}A]\!] = \{x \mid \forall y,\ \text{if } yRx \text{ then } y \in [\![A]\!]\}$.
- Clearly if $\Diamond A \rightarrow B$ is valid, then $A \rightarrow \Box^{\downarrow}B$ is valid as well.
- Conversely, if $A \rightarrow \Box^{\downarrow}B$ is valid, then $\Diamond A \rightarrow B$ must be as well.
- In **B**, **S5**, or other 'symmetric' modal logics, $\Diamond$ and $\Box$ already form a residuated pair, since $\Box$ and $\Box^{\downarrow}$ then coincide.

The Logic of Residuation

- Michael Moortgat introduced an extension of the Lambek Calculus NL to capture a residuated pair of 'modal' operators $\diamond$ and $\Box$.
- For this we extend the language of categorial grammar to include these new operators. Apart from atoms and formulas A/B and $A\backslash B$, we also have formulas $\diamond A$ and $\Box A$.
- Our structures now consist not only of sequences of formulas $(A \circ B)$, but also structures of the form $\langle A \rangle$.
- The natural deduction formulation is an extension of that for NL.

The Logic of Residuation

$$(\mathrm{Ax})\ \frac{}{A \blacktriangleright A} \qquad \frac{\Delta \blacktriangleright B \qquad \Gamma[B] \blacktriangleright A}{\Gamma[\Delta] \blacktriangleright A}\ (\mathrm{cut})$$

$$(/E)\ \frac{\Delta \blacktriangleright A/B \qquad \Gamma \blacktriangleright B}{(\Delta \circ \Gamma) \blacktriangleright A} \qquad \frac{\Gamma \blacktriangleright B \qquad \Delta \blacktriangleright A\backslash B}{(\Gamma \circ \Delta) \blacktriangleright A}\ (\backslash E)$$

$$(/I)\ \frac{(\Delta \circ B) \blacktriangleright A}{\Delta \blacktriangleright A/B} \qquad \frac{(B \circ \Delta) \blacktriangleright A}{\Delta \blacktriangleright A\backslash B}\ (\backslash I)$$

$$(\Diamond E)\ \frac{\Delta \blacktriangleright \Diamond A \qquad \Gamma[\langle A \rangle] \blacktriangleright B}{\Gamma[\Delta] \blacktriangleright B} \qquad \frac{\Gamma \blacktriangleright \Box A}{\langle \Gamma \rangle \blacktriangleright A}\ (\Box E)$$

$$(\Diamond I)\ \frac{\Gamma \blacktriangleright A}{\langle \Gamma \rangle \blacktriangleright \Diamond A} \qquad \frac{\langle \Gamma \rangle \blacktriangleright A}{\Gamma \blacktriangleright \Box A}\ (\Box I)$$

This system is referred to as $\mathrm{NL}(\Diamond)$.

$\Diamond$ and $\Box$ as residuated pair

$\Diamond A \blacktriangleright B$ implies $A \blacktriangleright \Box B$:

$$\dfrac{\dfrac{\dfrac{\Diamond A \blacktriangleright B \qquad \dfrac{\dfrac{}{A \blacktriangleright A}\,(\mathrm{Ax})}{\langle A \rangle \blacktriangleright \Diamond A}\,(\Diamond I)^1}{\langle A \rangle \blacktriangleright B}\,(\mathrm{cut})}{A \blacktriangleright \Box B}\,(\Box I)}{}$$

$A \blacktriangleright \Box B$ implies $\Diamond A \blacktriangleright B$:

$$\dfrac{(\Box E)\,\dfrac{A \blacktriangleright \Box B}{\langle A \rangle \blacktriangleright B} \qquad \dfrac{}{\Diamond A \blacktriangleright \Diamond A}\,(\mathrm{Ax})}{\Diamond A \blacktriangleright B}\,(\Diamond E)$$

Capturing Monotonicity and Polarity

The following are rough glosses of how statements in NL($\Diamond$) correspond to information about monotonicity or polarity:

- $\Gamma \blacktriangleright A/B$ or $\Gamma \blacktriangleright A\backslash B$ — Γ is a monotone function.
- $\Gamma \blacktriangleright A/\Diamond B$ or $\Gamma \blacktriangleright A\backslash\Diamond B$ — Γ is an antitone function.
- $\Gamma \blacktriangleright \Box A$ — Γ *must* have polarity $-$.
- $\langle\Gamma\rangle$ — Γ has polarity $-$.
- $\Gamma \neq \langle\Gamma'\rangle$ — Γ has polarity $+$.

Capturing Monotonicity and Polarity

- In place of Sánchez-Valencia's marking algorithm, or Dowty's internalizing marking schema, Bernardi adds two structural rules to NL($\Diamond$) to compute final polarity:

$$(\mathsf{Pol}_1)\ \frac{\Delta[\langle\Gamma_1 \circ \Gamma_2\rangle] \blacktriangleright A}{\Delta[\langle\Gamma_1\rangle \circ \langle\Gamma_2\rangle] \blacktriangleright A}$$

$$(\mathsf{Pol}_2)\ \frac{\Delta[\langle\langle\Gamma\rangle\rangle] \blacktriangleright A}{\Delta[\Gamma] \blacktriangleright A}$$

- This will be important for distinguished NPI licensing and genuine antitonicity, when necessary.

A Small Lexicon

As before, with polarity information now marked with $\Diamond$:

- Theodore, np
- candidate, n
- likes, $(s\backslash np)/np$
- left, iv
- every, $\begin{cases} (s/(s\backslash np))/\Diamond n \\ (s\backslash(s/np))/\Diamond n \end{cases}$
- no, $\begin{cases} (s/\Diamond(s\backslash np))/\Diamond n \\ (s\backslash\Diamond(s/np))/\Diamond n \end{cases}$

And a negative polarity item:

- yet, $\Box\Diamond iv\backslash\Box\Diamond iv$

Examples 8 and 9

$$\dfrac{\text{Theodore} \blacktriangleright np \qquad \dfrac{\text{likes} \blacktriangleright iv/(s\backslash(s/np)) \qquad \dfrac{\text{every} \blacktriangleright (s\backslash(s/np))/\Diamond n \qquad \dfrac{\text{candidate} \blacktriangleright n}{\langle\text{candidate}\rangle \blacktriangleright \Diamond n}}{\text{every } \langle\text{candidate}\rangle \blacktriangleright s\backslash(s/np)}}{\text{likes every } \langle\text{candidate}\rangle \blacktriangleright iv}}{\text{Theodore likes every } \langle\text{candidate}\rangle \blacktriangleright s}$$

$$\dfrac{\dfrac{\text{no} \blacktriangleright (s/\Diamond iv)/\Diamond n \qquad \dfrac{\text{candidate} \blacktriangleright n}{\langle\text{candidate}\rangle \blacktriangleright \Diamond n}}{\text{no } \langle\text{candidate}\rangle \blacktriangleright s/\Diamond iv} \qquad \dfrac{\dfrac{\text{likes} \blacktriangleright iv/(s\backslash(s/np)) \qquad \text{Theodore} \blacktriangleright s\backslash(s/np)}{\text{likes Theodore} \blacktriangleright iv}}{\langle\text{likes Theodore}\rangle \blacktriangleright \Diamond iv}}{\text{no } \langle\text{candidate}\rangle\ \langle\text{likes Theodore}\rangle \blacktriangleright s}$$

Example 10

Example from Bernardi [1]:

$$
\dfrac{
  \dfrac{
    \text{no} \blacktriangleright (s/\Diamond iv)/\Diamond n
    \qquad
    \dfrac{\text{candidate} \blacktriangleright n}{\langle\text{candidate}\rangle \blacktriangleright \Diamond n}\,(\Diamond I)
  }{
    \text{no } \langle\text{candidate}\rangle \blacktriangleright s/\Diamond iv
  }
  \qquad
  (\Box E)\,\dfrac{
    \dfrac{
      (\Box I)\,\dfrac{(\Diamond I)\,\dfrac{\text{left} \blacktriangleright iv}{\langle\text{left}\rangle \blacktriangleright \Diamond iv}}{\text{left} \blacktriangleright \Box\Diamond iv}
      \qquad
      \text{yet} \blacktriangleright \Box\Diamond iv \backslash \Box\Diamond iv
    }{
      \text{left yet} \blacktriangleright \Box\Diamond iv
    }
  }{
    \langle\text{left yet}\rangle \blacktriangleright \Diamond iv
  }
}{
  \text{no } \langle\text{candidate}\rangle\ \langle\text{left yet}\rangle \blacktriangleright s
}
$$

N.B. These markings can also serve as basis of a monotonicity calculus.

Example 11

$$
\dfrac{\text{Theodore} \blacktriangleright np \qquad \dfrac{\text{doubts} \blacktriangleright (s\backslash np)/\Diamond s \qquad \dfrac{\dfrac{\text{anybody} \blacktriangleright \Box\Diamond s/\Box\Diamond iv \qquad \dfrac{\dfrac{\text{left} \blacktriangleright iv}{\langle\text{left}\rangle \blacktriangleright \Diamond iv}}{\text{left} \blacktriangleright \Box\Diamond iv}}{\text{anybody left} \blacktriangleright \Box\Diamond s}}{\langle\text{anybody left}\rangle \blacktriangleright \Diamond s}}{\text{doubts } \langle\text{anybody left}\rangle \blacktriangleright s\backslash np}}{\text{Theodore doubts } \langle\text{anybody left}\rangle \blacktriangleright s}
$$

Supposing that 'if' is assigned category $(s/s)/\Diamond s$, we can now parse the problematic sentence correctly.

Example 11

$$
\dfrac{\dfrac{\text{if} \blacktriangleright (s/s)/\Diamond s \qquad \dfrac{\dfrac{\text{Theodore} \blacktriangleright np \qquad \dfrac{\text{doubts} \blacktriangleright (s\backslash np)/\Diamond s \qquad \dfrac{\dfrac{\text{anybody} \blacktriangleright \Box\Diamond s/\Box\Diamond iv \qquad \dfrac{\dfrac{\text{left} \blacktriangleright iv}{\langle\text{left}\rangle \blacktriangleright \Diamond iv}}{\text{left} \blacktriangleright \Box\Diamond iv}}{\text{anybody left} \blacktriangleright \Box\Diamond s}}{\langle\text{anybody left}\rangle \blacktriangleright \Diamond s}}{\text{doubts } \langle\text{anybody left}\rangle \blacktriangleright s\backslash np}}{\text{Theodore doubts } \langle\text{anybody left}\rangle \blacktriangleright s}}{\langle\text{Theodore doubts } \langle\text{anybody left}\rangle\rangle \blacktriangleright \Diamond s}}{\text{if } \langle\text{Theodore doubts } \langle\text{anybody left}\rangle\rangle \blacktriangleright s/s}}{\text{if } \langle\text{Theodore}\rangle\ \langle\text{doubts}\rangle \text{ anybody left} \blacktriangleright s/s}
$$

- There is clearly some deep connection between the distribution of negative polarity items and antitone contexts. But as we have seen with examples like *If Theodore doesn't like any candidate, he won't vote*, the connection is somewhat tenuous.

- Needless to say, no one ever thought or suggested NPIs would occur in exactly the antitone contexts. It is not easy to compute whether the NPI 'any good' is in an antitone context in an example like:

  No one who likes a candidate who doubts himself to be any good at ping pong would say such a thing.

- Given that bounded, finite agents like us are the ones uttering and comprehending expressions with NPIs, perhaps we should consider not *actual* monotonicity facts, but *perceived* monotonicity facts. This is more in the spirit of surface reasoning anyway.

- Such an idea is not new. For instance, van der Wouden wrote:

  "As a rule of thumb, everything that feels negative ... is monotone decreasing [and hence licenses NPIs]."

- In a paper from last year year Chemla, Homer, and Rothschild [3] investigate this idea from an experimental point of view.
- They are interested not only in the relation between antitonicity and NPIs, but also the relations between these and scalar implicatures. We focus here on their results concerning antitonicity and NPIs.

- 45 subjects, all native French speakers, were tested for their judgments on monotonicity inferences and for grammaticality of sentences with and without NPIs.

- For monotonicity inferences, subjects were asked whether one sentence follows from another, where one results from the other by predicate restriction:
  - Moins de 12 aliens ont goûté du saumon fumé.
    Moins de 12 aliens ont goûté du saumon.
  - *Fewer than 12 aliens tasted smoked salmon.*
    Fewer than 12 aliens tasted salmon.

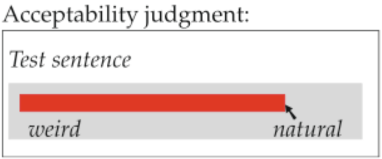
- For NPI grammaticality judgments, subjects were given two sentences on two sides of the screen, one with an NPI ('le moindre'), the other without, and asked to judge their grammaticality.
  - Chaque alien qui a de l'intérêt pour la littérature est rouge.
    Chaque alien qui a le moindre intérêt pour la littérature est rouge.
  - *Each alien who takes an interest in literature is red.*
    Each alien who takes the least interest in literature is red.

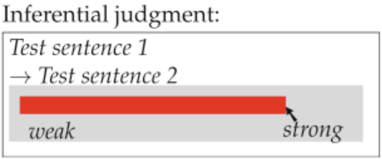
- Sentences were varied by subject matter (though all about aliens) and by quantifier used:

  - chaque (each)
  - certains (some)
  - aucun (no)
  - seulement (only)
  - moins de 12 (fewer than 12)
  - plus de 12 (more than 12)
  - exactement 42 (exactly 42)

  and by whether the predicate restriction, or NPI, occurs in the restrictor or the nuclear scope of the quantifier.

- Subjects were asked to give their judgments on a sliding scale:





Bare Results

	NPI	noNPI	log(NPI/noNPI)
NO$_S$	*81% (2.4)*	*87% (1.6)*	*-.192*
SOME$_S$	*18% (2.0)*	*94% (.73)*	*-2.42*
EVERY$_S$	*22% (2.4)*	*92% (1.1)*	*-2.26*
LESS THAN 12$_S$	*35% (2.7)*	*91% (1.1)*	*-1.57*
AT MOST 12$_S$	*26% (2.4)*	*85% (2.0)*	*-1.82*
EXACTLY 42$_S$	*20% (2.1)*	*91% (1.1)*	*-2.22*
ONLY 12$_S$	*29% (2.5)*	*92% (1.0)*	*-1.81*

(N.B. Is 'le moindre' a *strong* NPI?)

	MonD	MonU
NO$_S$	*71.79%*	*39.69%*
SOME$_S$	*48.88%*	*79.32%*
ALL$_S$	*27.29%*	*83.46%*
LESS THAN 12$_S$	*66.72%*	*45.82%*
AT MOST 12$_S$	*55.92%*	*51.12%*
EXACTLY 42$_S$	*23.30%*	*58.69%*
ONLY 12$_S$	*27.38%*	*43.94%*

NPIs and Monotonicity

- To determine the extent to which monotonicity/antitonicity is a good predictor of NPI distribution, Chemla et al. compute a correlation coefficient (r^2) between the judgments of each subject individually. The mean values are:
  - NPIs / monotonicity: 23.2%.
  - NPIs / antitonicity: 28.1%.
  - NPIs / monotonicity and antitonicity combined: 45.8%

- Most interestingly, to determine whether perceived monotonicity or some shared distributed notion of monotonicity is at work, they also compute a score corresponding to the frequency of cases where an individual subject's judgments leads to a better predictor than those of other participants. The mean values are:
  - NPIs / monotonicity: 59%.
  - NPIs / antitonicity: 60%.
  - NPIs / monotonicity and antitonicity combined: 60%
- If subjective judgments were no better than the population's judgments these values would average out to around 50%.

Summary

- There is clearly a deep connection between negative polarity and NPIs and antitonicity. But it is not (exactly) the logician's notion of 'following from' that is relevant, at least not if we want to predict the actual distribution of NPIs.
- Dowty, Moss, and Bernardi have offered some very elegant logical systems to capture aspects of NPI distribution. Is it possible to adapt these systems to come closer to 'perceived monotonicity'?
- Danescu et al. demonstrated some attractive practical applications of this connection, and Chemla et al. offered some empirical studies targeting a more precise characterization of the connection. But it is clear there is much work to be done from these angles as well.

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